Calculus I: Additional Review Integration Name ______________________

Evaluate the given indefinite integral:

1. $\int (6x^2 - 9x + 3)\, dx$

$2x^3 - \frac{9}{2}x^2 + 3x + c$

2. $\int 3x\sqrt{2x^2+3}\, dx$

$u = 2x^2 + 3$

$\frac{du}{dx} = 4x$

$\frac{du}{4x} = dx$

$\int 3xu^{1/2} \frac{du}{4x}$

$\int \frac{3}{4} u^{1/2} du$

$\frac{2}{3} \cdot \frac{3}{4} u^{3/2} + c$

$\frac{1}{2}(2x^2+3)^{3/2} + c$

3. $\int -4\sec(x)\tan(x)\, dx$

$-4\sec x + c$

4. $\int \frac{2x - 3\sqrt{x} + 5}{\sqrt[3]{x}}\, dx$

$\int \frac{2x - 3x^{1/2} + 5}{x^{1/3}} dx$

$\int \left(2x^{2/3} - 3x^{1/6} + 5x^{-1/3}\right) dx$

$\frac{6}{5}x^{5/3} - \frac{18}{7}x^{7/6} + \frac{15}{2}x^{2/3} + c$

5. $\int 4x^2 \csc^2(2x^3)\, dx$

$u = 2x^3$

$\frac{du}{dx} = 6x^2$

$\frac{du}{6x^2} = dx$

$\int 4x^2 \csc^2(u) \frac{du}{6x^2}$

$\int \frac{2}{3} \csc^2(u)\, du$

$-\frac{2}{3}\cot(u) + c$

$-\frac{2}{3}\cot(2x^3) + c$

Challenge Problem

6. $\int \sec^3(x)\tan(x)\, dx$

$u = \sec(x)$

$\frac{du}{dx} = \sec(x)\tan(x)$

$\frac{du}{\sec(x)\tan(x)} = dx$

$\int \sec^2(x)\sec(x)\tan(x)\, dx$

$\int u^2 \sec(x)\tan(x) \frac{du}{\sec(x)\tan(x)}$

$\int u^2 du$

$\frac{1}{3}u^3 + c$

$\frac{1}{3}\sec^3(x) + c$

7. $\int \frac{12x - 10}{\sqrt{3x^2 - 5x}}\, dx$

$u = 3x^2 - 5x$

$\frac{du}{dx} = 6x - 5$

$\frac{du}{6x-5} = dx$

$\int (12x-10)(3x^2-5x)^{-1/2} dx$

$\int 2(6x-5)(u)^{-1/2} \frac{du}{6x-5}$

$\int 2u^{-1/2} du$

$4u^{1/2} + c$

$4(3x^2-5x)^{1/2} + c$

8. $\int \frac{1}{\sqrt{x}} \cos(\sqrt{x})\, dx$

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{\frac{1}{2}x^{-1/2}} = dx$

$\int x^{-1/2} \cos(x^{1/2})\, dx$

$\int x^{-1/2} \cos(u) \frac{du}{\frac{1}{2}x^{-1/2}}$

$\int 2\cos(u)\, du$

$2\sin(u) + c$

$2\sin(x^{1/2}) + c$

9. $\int dx$

x + c

Evaluate the definite integral (non-calculator):

10. $\int_1^3 \left(3x^2 - 8x - 2\right) dx$

$$x^3 - 4x^2 - 2x\Big|_1^3$$
$$F(3) - F(1)$$
$$\left((3)^3 - 4(3)^2 - 2(3)\right) - \left((1)^3 - 4(1)^2 - 2(1)\right)$$
$$(-15) - (-5)$$
$$-10$$

11. $\int_0^{\frac{3\pi}{2}} 2\cos(x)\, dx$

$$2\sin(x)\Big|_0^{\frac{3\pi}{2}}$$
$$F\left(\frac{3\pi}{2}\right) - F(0)$$
$$\left(2\sin\left(\frac{3\pi}{2}\right)\right) - \left(2\sin(0)\right)$$
$$\left(2(-1)\right) - \left(2(0)\right)$$
$$-2$$

Evaluate the definite integral (calculator may be used after integral is taken):

12. $\int_0^1 3x\sqrt{2x^2+9}\, dx$

$$u = 2x^2 + 9$$
$$\frac{du}{dx} = 4x$$
$$\frac{du}{4x} = dx$$
$$u = 2(1)^2 + 9 = 11$$
$$u = 2(0)^2 + 9 = 9$$
$$\int_9^{11} 3x(u)^{1/2} \frac{du}{4x}$$
$$\int_9^{11} \frac{3}{4} u^{1/2} du$$
$$\frac{1}{2} u^{3/2}\Big|_9^{11}$$
$$F(11) - F(9)$$
$$4.741$$

13. $\int_\pi^{\frac{5\pi}{2}} x^2 \cos(3x^3)\, dx$

$$u = 3x^3$$
$$\frac{du}{dx} = 9x^2$$
$$\frac{du}{9x^2} = dx$$
$$u = 3\left(\frac{5\pi}{2}\right)^3 = \frac{375\pi^3}{8}$$
$$u = 3(\pi)^3 = 3\pi^3$$
$$\int_{3\pi^3}^{\frac{375\pi^3}{8}} x^2 \cos(u) \frac{du}{9x^2}$$
$$\int_{3\pi^3}^{\frac{375\pi^3}{8}} \frac{1}{9}\cos(u)\, du$$
$$\frac{1}{9}\sin(u)\Big|_{3\pi^3}^{\frac{375\pi^3}{8}}$$
$$F\left(\frac{375\pi^3}{8}\right) - F\left(3\pi^3\right)$$
$$.206$$

14. $\int_2^5 \left(3\sqrt{x} - \frac{2}{x^3}\right) dx$

$$\int_2^5 \left(3x^{1/2} - 2x^{-3}\right) dx$$
$$2x^{3/2} + x^{-2}\Big|_2^5$$
$$F(5) - F(2)$$
$$16.494$$

Approximate the area bound by the curve and the *x*-axis in the given interval using right and left end-point approximations with the given *n* value.

15. $f(x) = 3 + \sqrt{x} \quad [0, 6]; \quad n = 20$

Left: $sum\left(seq\left(\left(3+\sqrt{x}\right)\frac{3}{10}, x, 0, 6 - \frac{3}{10}, \frac{3}{10}\right)\right)$ = 27.398

Right: $sum\left(seq\left(\left(3+\sqrt{x}\right)\frac{3}{10}, x, 0 + \frac{3}{10}, 6, \frac{3}{10}\right)\right)$ = 28.133

Average : $\frac{27.398 + 28.133}{2}$ =27.766

16. $f(x) = 2x^2 + 5x - 1 \quad [2.1, 4.3]; \quad n = 18$

Left: $sum\left(seq\left(\left(2x^2 + 5x - 1\right)\frac{2.2}{18}, x, 2.1, 4.3 - \frac{2.2}{18}, \frac{2.2}{18}\right)\right)$ = 77.449

Right: $sum\left(seq\left(\left(2x^2 + 5x - 1\right)\frac{2.2}{18}, x, 2.1 + \frac{2.2}{18}, 4.3, \frac{2.2}{18}\right)\right)$ = 82.235

Average : $\frac{77.449 + 82.235}{2}$ =79.842

Find the exact area bound by the given function in the given interval. Sketch the graph and shade the area you are finding:

17. $f(x)=3x^2-4x+1 \quad [1,3]$

$$\int_1^3 \left(3x^2-4x+1\right)dx$$

$$x^3-2x^2+x\Big|_1^3$$

$$F(3)-F(1)$$

$$12$$

18. $f(x)=x^2+3x-4 \quad [0,2]$

$$\left|\int_0^1 \left(x^2+3x-4\right)dx\right|+\int_1^2 \left(x^2+3x-4\right)dx$$

$$\left|\frac{1}{3}x^3-\frac{3}{2}x^2-4x\Big|_0^1\right|+\frac{1}{3}x^3-\frac{3}{2}x^2-4x\Big|_1^2$$

$$\left|F(1)-F(0)\right|+F(2)-F(1)$$

$$\left|-2.167\right|+2.833$$

$$5$$

Find *f*(*x*) given the following information:

19. $f'(x)=6x-5$
$f(1)=4$

$$\int (6x-5)dx$$

$$f(x)=3x^2-5x+c$$

$$4=3(1)^2-5(1)+c$$

$$4=3-5+c$$

$$4=-2+c$$

$$6=c$$

$$f(x)=3x^2-5x+6$$

20. $f'(x)=3x\left(x^2-4\right)^3$
$f(2)=-7$

$$u=x^2-4$$

$$\frac{du}{dx}=2x$$

$$\frac{du}{2x}=dx$$

$$\int \left(3x\left(x^2-4\right)^3\right)dx$$

$$\int 3x(u)^3\frac{du}{2x}$$

$$\int \frac{3}{2}u^3du$$

$$\frac{3}{8}u^4+c$$

$$y=\frac{3}{8}\left(x^2-4\right)^4+c$$

$$-7=\frac{3}{8}\left((2)^2-4\right)^4+c$$

$$-7=c$$

$$y=\frac{3}{8}\left(x^2-4\right)^4-7$$

Given *F*(*x*), find *F*'(*x*):

21. $F(x)=\int_3^{4x^2}\left(t^2-5t\right)dt$

$$F'(x)=\left(\left(4x^2\right)^2-5\left(4x^2\right)\right)(8x)$$

22. $F(x)=\int_{3x}^{\cos(x^2)}\tan(2x)\,dx$

$$F'(x)=\tan\left(2\cos\left(x^2\right)\right)\left(-\sin\left(x^2\right)(2x)\right)-\tan\left(2(3x)\right)(3)$$

Find the average value of the given function on the given interval. Find the x value at which the average value occurs:

23. $f(x)=x^2-3x \quad [1,3]$

$$\int_1^3 \left(x^2-3x\right)dx = f(c)(3-1)$$
$$\frac{1}{3}x^3-\frac{3}{2}x^2 = f(c)(2)$$
$$F(3)-F(1) = f(c)(2)$$
$$\frac{-3.333}{2} = \frac{f(c)(2)}{2}$$
$$-1.667 = f(c)$$

$$-1.667 = x^2-3x$$
$$x = 2.264$$

24. $f(x)=-3\sin(x) \quad \left[0,\frac{\pi}{2}\right]$

$$\int_0^{\frac{\pi}{2}} \left(-3\sin(x)\right)dx = f(c)\left(\frac{\pi}{2}-0\right)$$
$$3\cos(x)\Big|_0^{\frac{\pi}{2}} = f(c)\left(\frac{\pi}{2}\right)$$
$$F\left(\frac{\pi}{2}\right)-F(0) = f(c)\left(\frac{\pi}{2}\right)$$
$$\frac{-3}{\frac{\pi}{2}} = \frac{f(c)\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}}$$
$$-1.91 = f(c)$$

$$-1.91 = -3\sin x$$
$$x = .69$$

25. $f(x)=\frac{2}{\sqrt{x}} \quad [1,4]$

$$\int_1^4 \left(2x^{-\frac{1}{2}}\right)dx = f(c)(4-1)$$
$$4x^{\frac{1}{2}}\Big|_1^4 = f(c)(3)$$
$$F(4)-F(1) = f(c)(3)$$
$$\frac{4}{3} = \frac{f(c)(3)}{3}$$
$$1.333 = f(c)$$

$$1.333 = 2x^{-\frac{1}{2}}$$
$$x = 2.251$$

26. $f(x)=-x^2+11 \quad [2,5]$

$$\int_2^5 \left(-x^2+11\right)dx = f(c)(5-2)$$
$$-\frac{1}{3}x^3+11x\Big|_2^5 = f(c)(3)$$
$$F(5)-F(2) = f(c)(3)$$
$$\frac{-6}{3} = \frac{f(c)(3)}{3}$$
$$-2 = f(c)$$

$$-2 = -x^2+11$$
$$x = 3.606$$